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A Simple Linear and Iterative LMS Algorithm for Fundamental Matrix Estimating

José A. de França[1*], **Maria B. de M. França**[1], **Robinson Hoto**[2], **M. R. Stemmer**[3]

[1]Departamento de Engenharia Elétrica - Universidade Estadual de Londrina
Caixa Postal 6025 - 86051-990, Londrina, PR, Brazil

[2]Departamento de Matemática - Universidade Estadual de Londrina
Caixa Postal 6001 - 86051-990, Londrina, PR, Brazil

[3]Departamento de Automação e Sistemas - Universidade Federal de Santa Catarina
Caixa Postal 476 - 88040-970, Florianópolis, SC, Brazil

{josealexandre,bernadete}@eeol.org,hoto@uel.br,marcelo@das.ufsc.br

Abstract. *This paper deals with one of the fundamental problems on stereovision, that is, the estimation of the fundamental matrix from a set of matched points. To solve this, we propose an iterative but linear algorithm that is a simple minimization problem with only three unknowns. Besides this, we don't use Newton-type optimizers to solve it. Experimental results shown that the new technique has a good performance comparing to the non-linear ones, additionally it has a low computational cost and a fast convergence.*

1. Introduction

Two images captured by a stereovision system are related for the epipolar geometry (Faugeras and Luong, 2001). This geometry is completely characterized by a 3×3 matrix, called essential matrix (Longuet-Higgins, 1981) when the system is calibrated. It can be easily known from the camera projection matrices.

As result of the know-how acquired in the stereovision and epipolar geometry areas in the 90's, many researchers proposed new algorithms to camera self-calibration or uncalibrated stereovision systems. In these cases, the epipolar geometry is described by the fundamental matrix, the uncalibrated analogue of the essential matrix. It can be estimated from an initial set of matches.

The first algorithm to find the epipolar geometry from a set of point correspondences was proposed by Hesse (1863). Years later, it was improved by Sturm (1869). This method uses seven matches (the least possible), but it is very sensitive to noise. So, it hasn't practical application. About hundred years later, the 8-point method was proposed from the work of Longuet-Higgins (1981). It is a simple and direct method, that uses redundance trying to reduce the noise influence. However, it doesn't give good results in the noise presence. Even so, it was used as a tool for generation of initial estimates for iterative methods.

Hartley (1997) proposed a normalization that improves the 8-point algorithm's accuracy when it is applied to initial set of matches. After that normalization, the centroid of the points is at the origin and the average distance of a point from the origin is equal to $\sqrt{2}$.

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One disadvantage of 8-point algorithm is to neglect the rank 2 constraint (Faugeras and Luong, 2001). So, this constraint has to be imposed *a posteriori*. The most popular method is to replace the estimated matrix, $\mathbf{F}$, by the matrix $\widehat{\mathbf{F}}$ which minimizes the Frobenius norm of $\mathbf{F} - \widehat{\mathbf{F}}$, subject to $\det \widehat{\mathbf{F}} = 0$ (Zhang, 1998, Sect 3.2.3). Besides not embedding the rank 2 constraint, the Hartley's method isn't optimum, since all the entries of $\mathbf{F}$ do not have equal importance, and indeed some entries are more tightly constrained by the matches than others (Zhang, 1998; Hartley and Zisserman, 2000). So, several authors have proposed non-linear and iterative methods to calculate the fundamental matrix (Bartoli and Sturm, 2004; Chesi et al., 2002; Zhang and Loop, 2001). They have success because they obtain results better than the ones found by 8-point algorithm, but at a high computational cost. In fact, nonlinearly estimating the fundamental matrix suffers from the lack of a simple technique to represent it efficiently.

In this work, we propose an iterative and linear algorithm for fundamental matrix estimating. It is a simple minimization problem with only three unknowns. To solve this problem, we don't use Newton-type optimizers. Besides this, an approach more simple is used. By this way, good results are reached with a high efficient due the low computational cost and a fast convergence.

In the section 2 is presented the mathematical notation used in this paper. After that, the theory used to obtain the method proposed is presented in the section 3. Our method is developed in the section 4. The experimental results are presented in the section 5 and the conclusions in the section 6.

2. Notation

Matrices are represented by bold characters (letters, numbers or symbols) and constants by italic characters (letters, numbers or symbols). Then, considering the model of pinhole camera (Faugeras and Luong, 2001), the coordinates of a 3D point in the coordinates system of scene are presented as $\mathbf{M} = [x, y, z]^T$ and the correspondent point in the retinal image as $\mathbf{m} = [u, v]^T$. The homogeneous coordinates of a point $\mathbf{x} = [x, y, ...]^T$ are represented by $\widetilde{\mathbf{x}}$, i.e, $\widetilde{\mathbf{x}} = [x, y, ..., t]^T$. Finally, is used the practical notation $A^{-T} = (A^{-1})^T = (A^T)^{-1}$ to all invertible square matrix.

3. Fundamental matrix estimation

It is well known that the fundamental matrix (Faugeras and Luong, 2001; Hartley and Zisserman, 2000) concentrates all the geometric restrictions behind any pair of images, if those images are projections (in different perspectives) of the same scene. Besides, that matrix has dimension 3×3, rank 2 and it satisfies the following fundamental equation:

$$\widetilde{\mathbf{m}}'^T \mathbf{F} \widetilde{\mathbf{m}} = 0, \tag{1}$$

where $\widetilde{\mathbf{m}}$ and $\widetilde{\mathbf{m}}'$ are the projections of the same point $\mathbf{M}$ (in 3D projective space) in the image planes I and I' respectively (see figure 1).

From equation (1), it is observed that the fundamental matrix could be estimated from a set of matches $(\widetilde{\mathbf{m}}_i, \widetilde{\mathbf{m}}'_i)$ (see figure 1). One of the methods more popular to make that estimation is the use of the least-square algorithm to solve the following equation

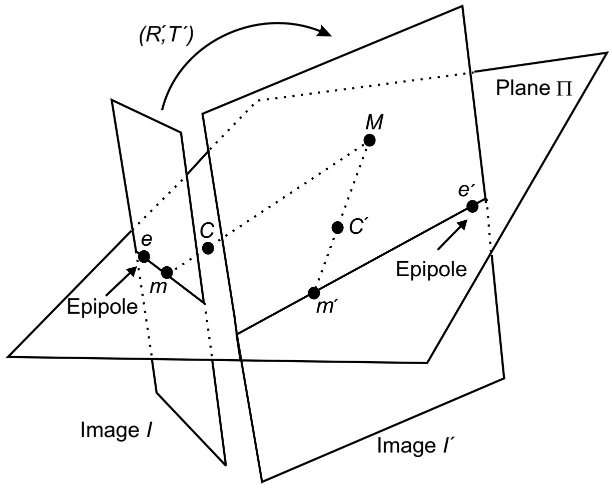


Figure 1: Epipolar geometry.

$$\sum_{i=1}^{n}(\widetilde{\mathbf{m}}_i'^T\mathbf{F}\widetilde{\mathbf{m}}_i)^2 = 0, \qquad \text{subject to } \|\mathbf{F}\| = 1, \tag{2}$$

where n is the number of matches.

This method is known as 8-point algorithm and it produces good results if the set of matches is normalized as suggested by Hartley (1997).

Other technique to estimate the fundamental matrix is the Gradient-Based Algorithm. That technique is based on observation that the variance of each sum's term of equation (2) is different. As the least-square algorithm produces the optimum solution only when those variances are equals, an alternative that produces better results is to solve

$$\sum_{i=1}^{n}\sigma_i^{-2}(\widetilde{\mathbf{m}}_i'^T\mathbf{F}\widetilde{\mathbf{m}}_i)^2 = 0, \qquad \text{subject to } \|\mathbf{F}\| = 1, \tag{3}$$

where σ_i is the variance of $\widetilde{\mathbf{m}}_i'^T\mathbf{F}\widetilde{\mathbf{m}}_i$. Now, the variance of each term of the equation (3) is equal to 1.

Considering a gaussian model of variance σ and average zero to the noise, we have (Faugeras and Luong, 2001; Zhang, 1998)

$$\sigma_i = \sigma(\mathbf{l}_i^T\mathbf{Z}\mathbf{l}_i + \mathbf{l}_i'^T\mathbf{Z}\mathbf{l}_i'), \tag{4}$$

where was considered $\mathbf{Z} = \text{diag}(1, 1, 0)$, $\mathbf{l}_i' = \mathbf{F}\widetilde{\mathbf{m}}_i$ and $\mathbf{l}_i = \mathbf{F}^T\widetilde{\mathbf{m}}_i'$.

Replacing (4) in (3), we have

$$\sum_{i=1}^{n}\frac{(\widetilde{\mathbf{m}}'^T\mathbf{F}\widetilde{\mathbf{m}})^2}{\mathbf{l}_i^T\mathbf{Z}\mathbf{l}_i + \mathbf{l}_i'^T\mathbf{Z}\mathbf{l}_i'} = 0, \qquad \text{subject to } \|\mathbf{F}\| = 1. \tag{5}$$

The equation 5 was obtained by Hartley and Zisserman (2000) when the Sampson distance (Sampson, 1982) was calculated to the case of equation (2).

The solution of problem (3) requires a parametrization of the fundamental matrix which enforces the rank 2 constraint (Hartley and Zisserman, 2000). A possible parametrization was proposed by Luong et al. (1993). In this, considering the epipoles (see figure 1) $\mathbf{e} = [e_u, e_v]^T$ and $\mathbf{e}' = [e'_u, e'_v]^T$ in the images I and I' respectively, $\mathbf{F}$ is written as

$$\mathbf{F} = \begin{bmatrix} a & b & -ae_u - be_v \\ c & d & -ce_u - de_v \\ -ae'_u - ce'_v & -be'_v - de'_v & F_{33} \end{bmatrix}, \tag{6}$$

where a, b, c and d are constants and $F_{33} = (ae_u + be_v)e'_u + (ce_u + de_v)e'_v$.

The Gradient-Based Algorithm was used with success in, e.g., (Zhang and Loop, 2001; Zhang, 1998; Torr and Zisserman, 1998). Unfortunately, the solution of the equation (5) is a non-linear problem. Generally, is used Newton-type optimizers to solve it. However, a new iterative but linear and extremely simple solution is proposed in the next section.

4. Proposed Method

Considering $\widetilde{\mathbf{m}} = [u_i, v_i, 1]^T$ and $\widetilde{\mathbf{m}}' = [u'_i, v'_i, 1]^T$, the equation (1) can be rewritten in the linear form as the following

$$\mathbf{a}_i^T\mathbf{f} = 0, \qquad \text{subject to } \|\mathbf{f}\| = 1, \tag{7}$$

where

$$\mathbf{a}_i = [u_iu'_i, v_iu'_i, u'_i, u_iv'_i, v_iv'_i, v'_i, u_i, v_i, 1]^T$$

(see (Hartley, 1997) to more details) and

$$\mathbf{f} = [F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33}]^T, \tag{8}$$

with F_{ij} is the element at i-th line and j-th column of fundamental matrix $\mathbf{F}$.

From (7), the equation (5) can be rewritten as

$$\sum_{i=1}^{n} w_i^2(\mathbf{a}_i^T\mathbf{f})^2 = 0, \qquad \text{subject to } \|\mathbf{f}\| = 1, \tag{9}$$

where

$$w_i = (\mathbf{l}_i^T\mathbf{Z}\mathbf{l}_i + \mathbf{l}_i'^T\mathbf{Z}\mathbf{l}'_i)^{-1/2}. \tag{10}$$

Recently, Liu and Männer (2003) has proposed an iterative solution to solve the equation (9). It was done considering $\mathbf{B} = [w_1\mathbf{a}_1, w_2\mathbf{a}_2, ..., w_n\mathbf{a}_n]^T$. So, the equation (9) can be written as

$$\|\mathbf{B}\mathbf{f}\|^2 = 0, \qquad \text{subject to } \|\mathbf{f}\| = 1. \tag{11}$$

Since that the matrix $\mathbf{B}$ also depends of $\mathbf{f}$, the method of Liu and Männer (2003), initially, considers $w_i = 1$ and it finds the vector $\mathbf{f}$ solving the equation (11) as a linear least-square problem. After, w_i (to $i = 1, 2, ..., n$) are iteratively updated and a best estimation of $\mathbf{f}$ is found to each step.

Although simple, the proposed approach by Liu and Männer (2003) has two problems:

(a) the equation (11) doesn't guarantee to find a rank 2 fundamental matrix. So, that constraint has to be imposed *a posteriori* (during each iteration step), and;

(b) as empirically demonstrated in the section 5, when the epipoles are away from the image's center, the method gives the same result of the normalized 8-point method (Hartley, 1997) (that is a non-iterative algorithm).

The rank 2 is one of mean characteristics of fundamental matrix. So, we conclude that the bad performance of method of (Liu and Männer, 2003) is caused meanly by absence of this constraint. Fortunately, we observe that it can be corrected considering that the fundamental matrix can be written as (Luong and Faugeras, 1996)

$$\mathbf{F} = [\widetilde{\mathbf{e}}']_\times \mathbf{H}, \tag{12}$$

where $\widetilde{\mathbf{e}}'$ is the epipole of $\mathbf{F}$, $[\widetilde{\mathbf{e}}']_\times$ is an antisymmetric matrix and $\mathbf{H}$ is a non-singular 3×3 matrix. Since that $\det([\widetilde{\mathbf{e}}']_\times) = 0$, the rank 2 constraint is automatically satisfied to all matrix $\mathbf{F}$ that satisfies (12).

From (12) and (7), the equation (9) can be rewritten as

$$\sum_{i=1}^{n} w_i^2 (\boldsymbol{\alpha}_i^T \widetilde{\mathbf{e}}')^2 = 0, \qquad \text{subject to } \|\widetilde{\mathbf{e}}'\| = 1, \tag{13}$$

where

$$\boldsymbol{\alpha}_i = \begin{bmatrix} H_{23} + H_{21}u_i - H_{33}v_i' - H_{31}u_iv_i' + H_{22}v_i - H_{32}v_i'v_i \\ -H_{13} + H_{33}u_i' - H_{11}u_i + H_{31}u_i'u_i - H_{12}v_i + H_{32}u_i'v_i \\ -H_{23}u_i' - H_{21}u_i'u_i + H_{13}v_i' + H_{11}u_iv_i' - H_{22}u_i'v_i + H_{12}v_i'v_i \end{bmatrix}, \tag{14}$$

with H_{ij} equal to the element of $\mathbf{H}$ at i-th line and j-th column.

From (13) and considering

$$\boldsymbol{\beta} = [w_1\alpha_1, w_2\alpha_2, ..., w_n\alpha_n]^T, \tag{15}$$

we can rewrite the problem (9) as

$$\|\boldsymbol{\beta}\widetilde{\mathbf{e}}'\|^2 = 0, \qquad \text{subject to } \|\widetilde{\mathbf{e}}'\| = 1. \tag{16}$$

Again, a non-iterative solution to the problem isn't possible, because $\boldsymbol{\beta}$ depends of the fundamental matrix. However, an iterative solution, without the use of non-linear methods based on Newton-type optimizers, is still viable. For this, we require an initial estimation, $\hat{\mathbf{F}}$, of the fundamental matrix. With $\hat{\mathbf{F}}$, we obtain the matrix $\mathbf{H}$ [equation (12)] and w_i, to $i = 1, 2, ...n$. So, we improve the estimative of the epipole $\widetilde{\mathbf{e}}'$ solving a simple problem of eigenvector. After that, from (12) and (10), we use w_i and obtain a new epipole estimation. This process continues iteratively until the wanted tolerance be reached.

The advantage of the proposed method is that the iterative part of algorithm consists of an extremely simple minimization problem. It involves the estimation of the three parameters (the homogeneous coordinates of the epipole $\mathbf{e}'$). In spite of that, the algorithm finds the fundamental matrix that minimizes the algebraic error for all matched points.

To obtain the matrix $\mathbf{H}$ from of the matrix $\hat{\mathbf{F}}$, we can use the following property valid to all vector $\mathbf{v}$

$$\|\mathbf{v}\|^2\mathbf{I} = \mathbf{v}\mathbf{v}^T - [\mathbf{v}]_\times^2. \tag{17}$$

Input: n matches $(\mathbf{m}_i, \mathbf{m}'_i)$ and an estimation, $\hat{\mathbf{F}}$, of the fundamental matrix.
Output: rank 2 fundamental matrix, $\mathbf{F}$, that satisfies the equation (5).

(i) Let $\tilde{\hat{\mathbf{m}}} = \mathbf{T}\tilde{\mathbf{m}}$ and $\tilde{\hat{\mathbf{m}}}' = \mathbf{T}'\tilde{\mathbf{m}}'$, where $\mathbf{T}$ and $\mathbf{T}'$ are 3×3 matrices that impose the proposed normalization by Hartley (1997);
(ii) Given the matrix $\hat{\mathbf{F}}$, calculate $\mathbf{H}$ with the equation (20);
(iii) Initialize the matrix $\boldsymbol{\beta}$ using the matches $(\hat{\mathbf{m}}_i, \hat{\mathbf{m}}'_i)$, making $w_i = 1$, and with the equations (14) and (15);
(iv) Compute the epipole $\tilde{\mathbf{e}}'$ solving the equation (16);
(v) Using the equation (12), update the estimation of $\hat{\mathbf{F}}$;
(vi) Using the matrix $\hat{\mathbf{F}}$ calculated in the previous step, update the matrix $\boldsymbol{\beta}$ considering

$$w_i = (\hat{\mathbf{l}}_i^T \mathbf{T}\mathbf{Z}\mathbf{T}^T \hat{\mathbf{l}}_i + \hat{\mathbf{l}}_i'^T \mathbf{T}'\mathbf{Z}\mathbf{T}'^T \hat{\mathbf{l}}'_i)^{-1/2},$$

where $\hat{\mathbf{l}}'_i = \hat{\mathbf{F}}\tilde{\hat{\mathbf{m}}}_i$ and $\hat{\mathbf{l}}_i = \hat{\mathbf{F}}^T \tilde{\hat{\mathbf{m}}}'_i$;
(vii) Compute the residue given by $\|\boldsymbol{\beta}\tilde{\mathbf{e}}'\|^2$;
(viii) Repeat the steps from (iv) to (vii) until the residue convergence, and;
(ix) Make $\mathbf{F} = \mathbf{T}'[\tilde{\mathbf{e}}']_\times \mathbf{H}\mathbf{T}$.

Algorithm 1: Simple linear and iterative LMS for fundamental matrix estimating.

So, from (17), we can write

$$\begin{aligned}\hat{\mathbf{F}} &= \frac{1}{\|\tilde{\mathbf{e}}'\|^2}(\tilde{\mathbf{e}}'\tilde{\mathbf{e}}'^T - [\tilde{\mathbf{e}}']_\times^2)\hat{\mathbf{F}} \\ &= \frac{1}{\|\tilde{\mathbf{e}}'\|^2}\tilde{\mathbf{e}}'\tilde{\mathbf{e}}'^{\mathbf{T}}\hat{\mathbf{F}} + [\tilde{\mathbf{e}}']_\times \left(-\frac{[\tilde{\mathbf{e}}']_\times}{\|\tilde{\mathbf{e}}'\|^2}\hat{\mathbf{F}}\right). \qquad (18)\end{aligned}$$

However, since that $\hat{\mathbf{F}}^T\tilde{\mathbf{e}}' = \tilde{\mathbf{e}}'^T\hat{\mathbf{F}} = 0$, the equation (18) is resumed to

$$\hat{\mathbf{F}} = [\tilde{\mathbf{e}}']_\times \left(-\frac{[\tilde{\mathbf{e}}']_\times}{\|\tilde{\mathbf{e}}'\|^2}\hat{\mathbf{F}}\right). \qquad (19)$$

Finally, we compare (19) and (12), we verify that

$$\mathbf{H} = -\frac{[\tilde{\mathbf{e}}']_\times}{\|\tilde{\mathbf{e}}'\|^2}\hat{\mathbf{F}}. \qquad (20)$$

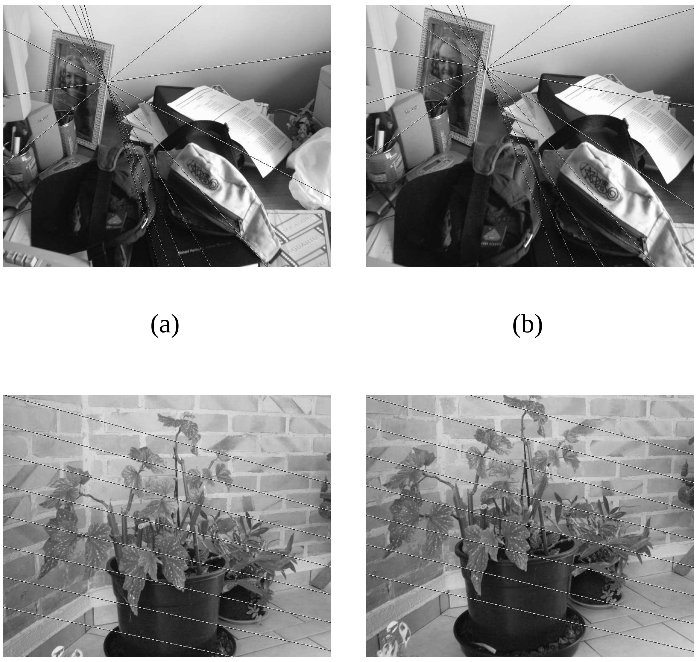
The algorithm 1 resumes the details of the iteration necessary to proposed method.

In the next section, the proposed algorithm is tested in real images. Besides, its accuracy is compared with the one the others methods.

5. Experimental Results

The proposed method was tested in several pairs of real images. Here, we present the results for two image pairs: the “*Desktop*” pair [figures 2(a) and 2(b)] that has epipoles close to image center, and; the “*Plant*” pair [figures 2(c) and 2(d)] with epipoles lie to infinite.

The performance of the proposed method (referenced in the ours results as **França**) was compared to three other methods: normalized 8-point method (referenced here as **Hartley 8-point**) (Hartley, 1997); method of Liu and Männer (2003) (referenced as **Liu and Männer**), and; gradient-based method (called **Gradient-Based**).



(a) (b)

(c) (d)

Figure 2: (a) and (b) form the "*Desktop*" image pair (with epipoles close to image center), and; (c) an (d) the "*Plant*" pair (epipoles lie to infinite).

The experiments were realized using the following methodology.

1. The matches were computed using the automatic technique described in (de França et al., 2004).
2. The fundamental matrix was estimated with different quantities of matched points. To each quantity, j, were selected 50 subsets (indexed by q) inside the total of N available correspondences. So, the fundamental matrix estimated in an assembly is referenced by $\mathbf{F}_{qj}$, where q is the number of assembly for a quantity of points indicated by j;
3. The relation between j and the number of matches, $n(j)$, is given by

$$n(j) = (j + 7), \qquad j = 1, ..., 63. \tag{21}$$

4. When necessary, was used the 8-point algorithm (Hartley, 1997) to give an initial estimation of fundamental matrix.
5. To each quantity of correspondence, j, we calculated the average of residue, $\overline{r}_j^2$, given by

$$\overline{r}_j^2 = \frac{1}{50N} \sum_{q=1}^{50} \sum_{i=1}^{N} d^2(\widetilde{\mathbf{m}}'_i, \mathbf{F}_j \widetilde{\mathbf{m}}_i) + d^2(\widetilde{\mathbf{m}}_i, \mathbf{F}_j^T \widetilde{\mathbf{m}}'_i). \tag{22}$$

This average was used as measurement of the quality of the estimated fundamental matrix.

The experiment results are presented in figures 3 and 4. From figures 3(a) and 3(c), we noted that the **França** method presents results very close to others obtained with the **Gradient-Based** method when few matched points are used. Otherwise, with few matched points, the **Liu and Männer** method has performance equal to **Hartley 8-point** method.

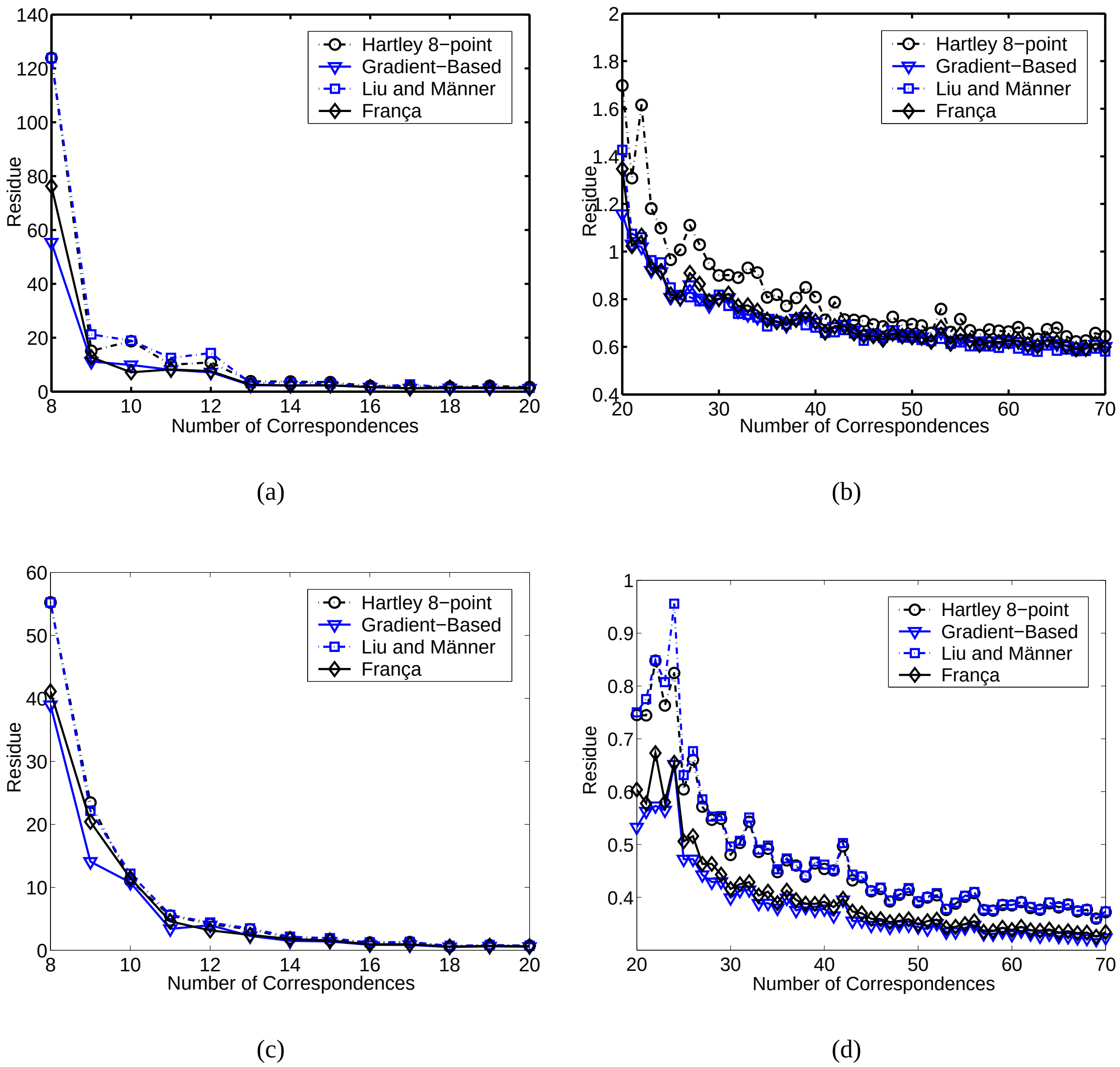


Figure 3: (a) and (b) are results of the "*Desktop*" image experiments to, respectively, small and big set of matched points, and; (c) and (d) are the results to "*Plant*" images.

When the number of matches used in the calculations increase, the behavior of the **Liu and Männer** and **França** algorithms are different, depending of the epipoles localization. To epipoles close to image centers, the **Liu and Männer**, **França** and **Gradient-Based** methods have very similar results [figure 3(b)]. However, to images with epipoles well away from the center, the **Liu and Männer** method has performance equal to **Hartley 8-point** method. The **França** method has intermediary performance between the **Hartley 8-point** and **Gradient-Based** methods [figure 3(d)].

Figure 4 presents the average of iteration number of the **Liu and Männer**, **França** and **Gradient-Based** methods. The **Liu and Männer** and **França** methods require two iterations in average. Otherwise, the **França** method requires a little more iterations when the epipoles lie to infinite. Already the **Gradient-Based** method, have required around 5 iterations.

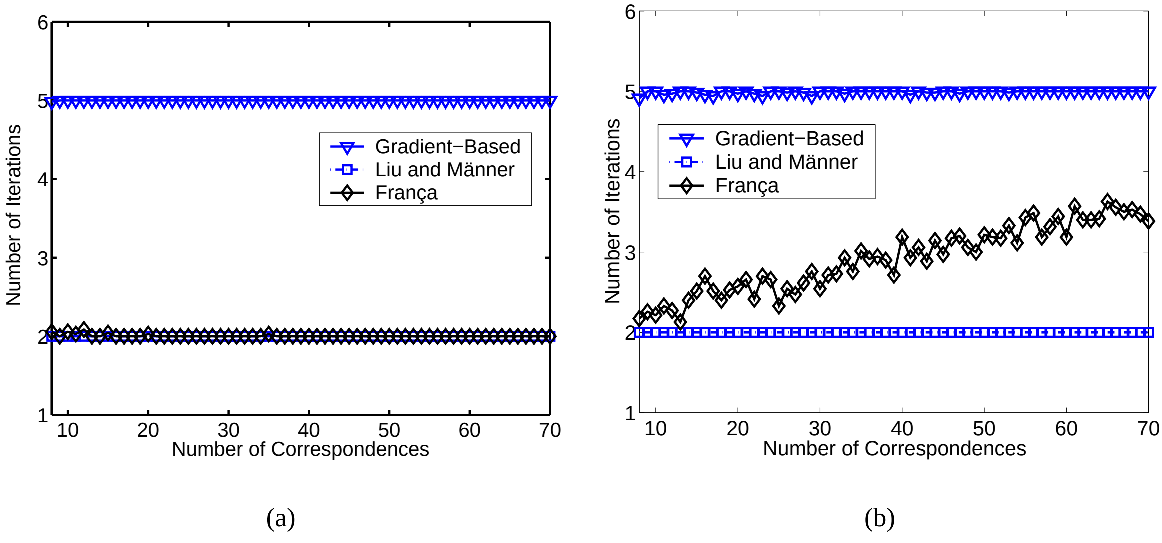


Figure 4: (a) and (b) present the iterations quantity of the experiments to, respectively, the "*Desktop*" and "*Plant*" images.

6. Conclusion

We have presented in this paper a new iterative but linear method to estimate the fundamental matrix. The method is a simple problem of minimization, where there are only three unknowns, and it produces explicitly a rank two fundamental matrix. Those improvements contributes to produce a fundamental matrix that better adjusts to set of matched points used in the calculations. Besides this, the method works well to images with epipoles lie to infinite or not.

Results in real images show that the proposed method has performance very closed to non-linear algorithms that use Newton-type optimizers. However, its performance is reached with smaller computational cost. So, it has to be used when we require better results than the ones obtained using Hartley's 8-point algorithm, but with a simple and fast solution.

Besides low computational cost and fast convergence, the proposed method presents a good performance, comparing to Hartley's 8-points method (Hartley, 1997), when we have only a small set of matches. So, it is ideal to use in algorithm that estimates the fundamental matrix robustly, for example, the LMedS (de França and Stemmer, 2003; Armangué and Salvi, 2003; Zhang, 1998). In these algorithms, the fundamental matrix is estimated several times with a very reduced number of matches.

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